Physics

Section – A

1. The time taken by an object to slide down 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is:

   (A) $1 - \dfrac{1}{n^2}$

   (B) $\sqrt{\dfrac{1}{1-n^2}}$

   (C) $\sqrt{1 - \dfrac{1}{n^2}}$

   (D) $1 + \dfrac{1}{n^2}$

2. Given below are two statements:

   Statement I: Electromagnetic waves are not deflected by electric and magnetic field.

   Statement II: The amplitude of electric field and the magnetic field in electromagnetic waves are related to each other as $E_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}\, B_0$.

   In the light of the above statements, choose the correct answer from the options given below:

   (A) Both Statement I and Statement II are false

   (B) Statement I is true but Statement II is flase

   (C) Statement I is flase but Statement II is true

   (D) Both Statement I and Statement II are true

3. A point charge 2×10^{-2} C is moved from P to S in a uniform electric field of 30 NC^{-1} directed along positive x-axis. If coordinates of P and S are (1, 2, 0) m and (0, 0, 0) m respectively, the work done by electric field will be

   (A) –600 mJ

   (B) 600 mJ

   (C) 1200 mJ

   (D) –1200 mJ

4. For the given logic gates combination, the correct truth table will be
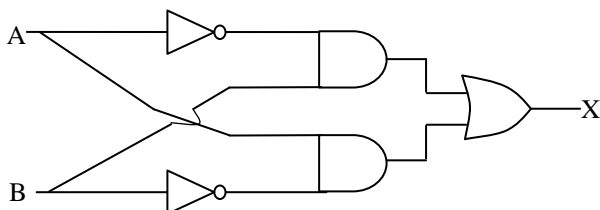
(A)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(B)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(C)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

(D)

| A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

5. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

   (A) Decrease the diameter of the objective lens

   (B) Increase the wave length of the light

   (C) Decrease the focal length of the eye piece.

   (D) Increase the refractive index of the medium between the object and objective lens

6. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\dfrac{1}{\sqrt{m}}$, the value of m is

   (A) 4

   (B) 2

   (C) 8

   (D) 16

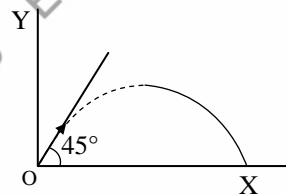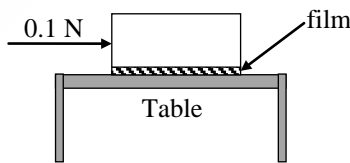7. For the given figures, choose the correct options:
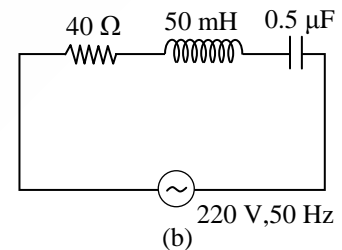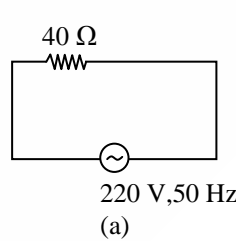
   (A) The rms current in circuit (b) can never be larger than that in (a)

   (B) The rms current in circuit (b) can be larger than that in (a)

   (C) At resonance, current in (b) is less than that in (a)

   (D) The rms current in figure (a) is laways equal to that in figure (b)

8. The equation of a circle is given by $x^2 + y^2 = a^2$, where a is the radius. If the equation is modified to change the origin other than (0, 0), then find out the correct dimensions of A and B in a new equation: $(x - At)^2 + \left(y - \dfrac{t}{B}\right)^2 = a^2$. The dimensions of t is given as $[T^{-1}]$.
   (A) A = [LT], B = $[L^{-1}T^{-1}]$
   (B) A = $[L^{-1}T^{-1}]$, B = [LT]
   (C) A = $[L^{-1}T]$, B = $[LT^{-1}]$
   (D) A = $[L^{-1}T^{-1}]$, B = $[LT^{-1}]$

9. Substance A has atomic mass number 16 and half life of 1 day. Another substance B has atomic mass number 32 and half life of $\dfrac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.
   (A) 3.38×10^{24}
   (B) 6.76×10^{24}
   (C) 6.76×10^{23}
   (D) 1.69×10^{24}

10. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be:
   (A) 16 T (B) 2 T (C) 4 T (D) 8 T

11. An object moves at a constant speed along a circular path in a horizontal plane with centre at the origin. When the object is at x = +2 m, its velocity is $-4\,\hat{j}$ m/s. The object's velocity (v) and acceleration (a) at x = –2 m will be
   (A) $v = 4\,\hat{i}$ m/s, $a = 8\,\hat{j}$ m/s^2
   (B) $v = -4\,\hat{j}$ m/s, $a = 8\,\hat{i}$ m/s^2
   (C) $v = -4\,\hat{i}$ m/s, $a = -8\,\hat{j}$ m/s^2
   (D) $v = 4\,\hat{j}$ m/s, $a = 8\,\hat{i}$ m/s^2

12. Heat energy of 184 kJ is given to ice of mass 600 g at –12°C. Specific heat of ice is 2222.3 J kg^{-1} °C^{-1} and latent heat of ice is 336 kJ/kg^{-1}
i.   Final temperature of system will be 0°C.
ii. Final temperature of the system will be greater than 0°C.
iii. The final system will have a mixture of ice and water in the ratio of 5 : 1.
iv. The final system will have a mixture of ice and water in the ratio of 1 : 5.
v.   The final system will have water only.
   Choose the correct answer from the options given below:
   (A) i and iv Only   (B) i and iii Only
   (C) i and v Only   (D) ii and iv Only

13. A force acts for 20 s on body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50 m in the next 10 s. the value of force will be:
   (A) 5 N   (B) 20 N
   (C) 10 N   (D) 40 N

14. At 300 K, the rms speed of oxygen molecules is $\sqrt{\dfrac{\alpha + 5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be
   (used $\pi = \dfrac{22}{7}$)
   (A) 27   (B) 24   (C) 32   (D) 28

15. With the help of potentiometer, we can determine the value of emf of a given cell. The sensitivity of the potentiometer is
i.   directly proportional to length of the potentiometer wire
ii. directly proportional to the potential gradient of the wire
iii. inversely proportional to the potential gradient of the wire
iv. inversely proportional to the length of the potentiometer wire
   Choose the correct option for the above statements:
   (A) iii only   (B) i and iii only
   (C) ii and iv only   (D) i only

16. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is-
   (A) 0.4   (B) 1.4   (C) 0.2   (D) 0.6

17. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 1080 km/h. If the density of air ρ is 1.2 kg m^{-3}, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be . (g = 10 m/s^2)
   (A) 8   (B) 6   (C) 16   (D) 10

18. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.
   (A) 12 hours   (B) 4 hours
   (C) 6 hours   (D) 3 hours

19. A square loop of area 25 cm^2 has a resistance of 10 Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be
   (A) 2.5×10^{-3} J   (B) 1.0×10^{-4} J
   (C) 1.0×10^{-3} J   (D) 5×10^{-3} J

20. Identify the correct statements from the following:
i   Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative.
ii. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket in negative.
iii. Work done by friction on a body sliding down an inclined plane is positive.
iv. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.
v.   Work done by the air resistance on an oscillating pendulum in negative.
Choose the correct answer from the options given below:
   (A)   ii and iv only   (B)   ii, iv and v only
   (C)   ii and v only   (D)   i and iii only

Section – B

Numerical Value Type Questions

21. An inductor of inductance 2 µH is connected in series with a resistance, a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $\dfrac{1}{x}$ F, where the value of x is_____. (Take $\pi = \dfrac{22}{7}$)

22. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is $\dfrac{x}{10} \times 10^{-3}$, where x is _____.

23. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium – 1) and 6.8 (medium – 2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1 + \dfrac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is _____.
(Given for dielectric media, $\mu_r = 1$)

24. A particle of mass 250 g executes a simple harmonic motion under a periodic force $F = (-25x)$ N. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is _____ cm.

25. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t(1 - e^{-\pi/2})$s. The value of t is _____.

26. When two resistances R_1 and R_2 connected in series and introduced into the left gap of a meter bridge and a resistance of 10Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3Ω is introduced into the right-gap to get null point at 40 cm from left end. The product of R_1R_2 is _____ Ω^2.

27. A particle of mass 100 g is projected at time $t = 0$ with a speed 20 ms^{-1} at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time $t = 2$s is found to be $\sqrt{K}$ kg m^2/s. The value of K is _____.
(Take $g = 10$ ms^{-2})

28. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V = 2ar^2 + b$.
Here, a and b are constant and r is the distance from the centre. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is _____.
ε = permittivity of the medium

29. A metal block of base area 0.20 m^2 is placed on a table, as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0×10^{-3} SI, the speed of block is _____ $\times 10^{-3}$ m/s.

30. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15Ω is used for shunting, null point moves to 300 cm. The internal resistance of the cell is _____ Ω.

Chemistry

Section – A

31. Correct order of spin only magnetic moment of the following complex ions is:
(Given At. no. Fe: 26, Co: 27)
(A) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$
(B) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$
(C) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$
(D) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

32. Match List I and List II

	List I		List II
i.	Osmosis	a.	Solvent molecules pass through semi permeable membrane towards solvent side.
ii.	Reverse osmosis	b.	Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charges electrodes.
iii.	Electro osmosis	c.	Solvent molecules pass through semi permeable membrane towards solution side.
iv.	Electrophoresis	d.	Dispersion medium moves in an electric field.

Choose the correct answer from the options given below:
(A) i – c, ii – a, iii – d, iv – b
(B) i – c, ii – a, iii – b, iv – d
(C) i – a, ii – c, iii – d, iv – b
(D) i – a, ii – c, iii – b, iv – d

33. Find out the major product for the following reaction.
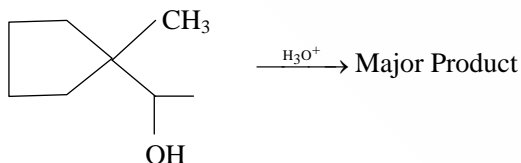
(A) (B)

(C) (D)

34. The concentration of dissolved Oxygen in water for growth of fish should be more than <u>X</u> ppm and Biochemical Oxygen Demand in clean water should be less than <u>Y</u> ppm. X and Y in ppm are, respectively.

(A)
X	Y
6	5

(B)
X	Y
6	12

(C)
X	Y
4	15

(D)
X	Y
4	8

35. Which of the following relations are correct?
i. $\Delta U = q + p\Delta V$
ii. $\Delta G = \Delta H - T\Delta S$
iii. $\Delta S = \dfrac{q_{rev}}{T}$
iv. $\Delta H = \Delta U - \Delta nRT$
(A) ii and iv Only
(B) iii and iv Only
(C) i and ii Only
(D) ii and iii Only

36. Reaction of propanamide with $Br_2/KOH_{(aq)}$ produces:
(A) Ethylamine
(B) Propylamine
(C) Propanenitrile
(D) Ethylnitrile

37. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator 'X' forms blue coloured complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are
(A) Methyl orange and iodine
(B) Starch and H_2O_2
(C) Methyl orange and H_2O_2
(D) Starch and iodine

38. Match **List I** and **List II**

	List I		List II
i.	van't Hoff factor, i	a.	Cryoscopic constant
ii.	k_f	b.	Isotonic solutions
iii.	Solutions with same osmotic pressure	c.	$\dfrac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
iv.	Azeotropes	d.	Solutions with same composition of vapour above it

Choose the correct answer from the options given below:
(A) i – a, ii – c, iii – b, iv – d
(B) i – c, ii – a, iii – b, iv – d
(C) i – c, ii – a, iii – d, iv – b
(D) i – c, ii – b, iii – a, iv – d

39. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A ?
(A) C_9H_6
(B) C_8H_6
(C) C_6H_6
(D) C_9H_9

40. The set of correct statements is:
i. Manganese exhibits +7 oxidation state in its oxide.
ii. Ruthenium and Osmium exhibit +8 oxidation in their oxides.
iii. Sc shows +4 oxidation state which is oxidizing in nature.
iv. Cr shows oxidising nature in +6 oxidation state.
    (A) i, ii and iv   (B) i and iii
    (C) ii and iii   (D) ii, iii and iv

41. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?
    (A) Hyperacidity
    (B) Anxiety and stress
    (C) Depression and hypertension
    (D) Stomach ulcers

42. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)
    (A) 2-Bromopentane
    (B) 2-Bromopropane
    (C) 1-Bromo-2-methylbutane
    (D) 2-Bromo-3,3-dimethylpentane

43. A solution of CrO_5 in amyl alcohol has a _____ colour.
    (A) Green   (B) Orange-Red
    (C) Yellow   (D) Blue

44. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are
    (A) 1, 3 and 3   (B) 1, 3 and 2
    (C) 1, 2 and 3   (D) 2, 3 and 3

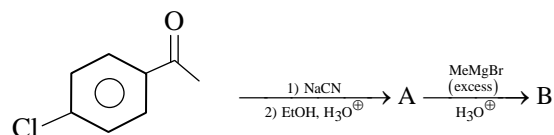45. Following tetrapeptide can be represented as
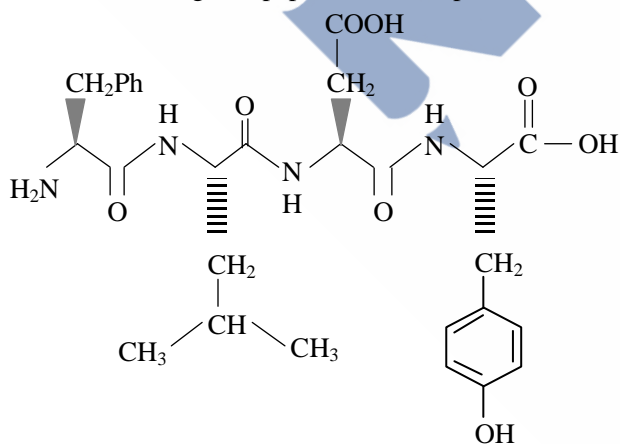
(F, L, D, Y, I, Q, P are one letter codes for amino acids)
    (A) FIQY   (B) PLDY
    (C) FLDY   (D) YQLF

46. The major component of which of the following ore is sulphide based mineral?
    (A) Sphalerite   (B) Malachite
    (C) Siderite   (D) Calamine

47. Find out the major products from the following reaction sequence.

(A)

(B)

(C)

(D)

48. Match **List I** and **List II**

	List I		List II
i.	Elastomeric polymer	a.	Urea formaldehyde resin
ii.	Fibre Polymer	b.	Polystyrene
iii.	Thermosetting Polymer	c.	Polyester
iv.	Thermoplastic Polymer	d.	Neoprene

Choose the correct answer from the options given below:
    (A) i – b, ii – c, iii – a, iv – d
    (B) i – d, ii – a, iii – c, iv – b
    (C) i – d, ii – c, iii – a, iv – b
    (D) i – b, ii – a, iii – d, iv – c

49. Given below are two statements:
    Statement I: The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.
    Statement II: The d orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below.
(A) Both the statements I and II are incorrect.
(B) Statement I is correct but statement II is incorrect.
(C) Statement I is incorrect but statement II is correct.
(D) Both the statements I and II are correct.

50. Given below are two statements:
Statement I: Nickel is being used as the catalyst for producing syn gas and edible fats.
Statement II: Silicon forms both electron rich and electron deficient hydrides.
In the light of the above statements, choose the most appropriate answer from the options given below:
(A) Both the statements I and II are incorrect.
(B) Statement I is incorrect but statement II is correct.
(C) Both the statements I and II are correct.
(D) Statement I is correct but statement II is incorrect.
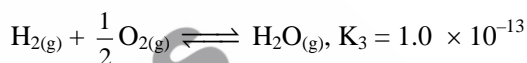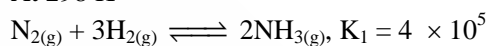
Section – B

Numerical Value Type Questions

51. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is _____ $\times 10^{21}$ (Nearest integer).
(Given $N_A = 6.02 \times 10^{23}$)

52. The denticity of the ligand present in the Fehling's reagent is _____.

53. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 Å. The radius of the third Bohr orbit of He^+ is _____ picometer.
(Nearest Integer)

54. Total number of acidic oxides among N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O and NO is _____.

55. The volume of HCl, containing 73 g L^{-1}, required to completely neutralise NaOH obtained by reacting 0.69 g of metallic sodium with water, is _____ mL. (Nearest Integer)
(Given: molar Masses of Na, Cl, O, H, are 23, 35.5, 16 and 1g mol^{-1} respectively)

56. The equilibrium constant for the reaction
$Zn_{(s)} + Sn^{2+}_{(aq)} \rightleftharpoons Zn^{2+}_{(aq)} + Sn_{(s)}$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^{\ominus}_{Zn^{2+}/Zn} = -0.76$ V is _____ $\times 10^{-2}$ V. (Nearest integer).
Given: $\dfrac{2.303\,RT}{F} = 0.059$ V

57. For conversion of compound A $\rightarrow$ B, the rate constant of the reaction was found to be 4.6×10^{-5} L mol^{-1} s^{-1}. The order of the reaction is _____.
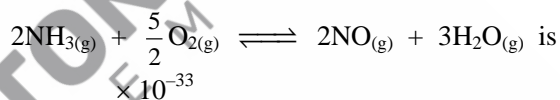
58. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is _____ g mol^{-1} (Nearest integer).

59. On heating, $LiNO_3$ gives how many compounds among the following? _____
Li_2O, N_2, O_2, $LiNO_2$, NO_2

60. At 298 K
$N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$, $K_1 = 4 \times 10^5$
$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$, $K_2 = 1.6 \times 10^{12}$
$H_{2(g)} + \dfrac{1}{2}O_{2(g)} \rightleftharpoons H_2O_{(g)}$, $K_3 = 1.0 \times 10^{-13}$
Based on above equilibria, the equilibrium constant of the reaction,
$2NH_{3(g)} + \dfrac{5}{2}O_{2(g)} \rightleftharpoons 2NO_{(g)} + 3H_2O_{(g)}$ is _____ $\times 10^{-33}$
(Nearest integer).

Mathematics

Section – A

61. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is
(A) 432
(B) 507
(C) 472
(D) 400

62. Let R be a relation defined on N as a R b if $2a + 3b$ is a multiple of 5, a, b $\in$ N. Then R is
(A) not reflexive
(B) transitive but not symmetric
(C) an equivalence relation
(D) symmetric but not transitive

63. The value of the integral $\displaystyle\int_{1}^{2}\left(\dfrac{t^4+1}{t^6+1}\right)$ dt is
(A) $\tan^{-1}2 + \dfrac{1}{3}\tan^{-1}8 - \dfrac{\pi}{3}$
(B) $\tan^{-1}\dfrac{1}{2} + \dfrac{1}{3}\tan^{-1}8 - \dfrac{\pi}{3}$
(C) $\tan^{-1}2 - \dfrac{1}{3}\tan^{-1}8 + \dfrac{\pi}{3}$
(D) $\tan^{-1}\dfrac{1}{2} - \dfrac{1}{3}\tan^{-1}8 + \dfrac{\pi}{3}$

64. The set of all values of t $\in$ R, for which the matrix $\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$ is invertible, is

    (A) R

    (B) $\left\{ k\pi + \dfrac{\pi}{4}, k \in Z \right\}$

    (C) $\{k\pi, k \in Z)$

    (D) $\left\{ (2k+1)\dfrac{\pi}{2}, k \in Z \right\}$

65. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is:

    (A) $\dfrac{3}{2}\sqrt{5}$

    (B) $\dfrac{9}{\sqrt{5}}$

    (C) $3\sqrt{5}$

    (D) $5\sqrt{3}$

66. If $\vec{a} = \hat{i} + 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. Then $\vec{r} \cdot \vec{c}$ is equal to

    (A) 34 (B) 30
    (C) 36 (D) 32

67. Let $y = y(x)$ be the solution of the differential equation $x\log_e x \dfrac{dy}{dx} + y = x^2 \log_e x$, $(x > 1)$. If $y(2) = 2$, then y (e) is equal to

    (A) $\dfrac{1 + e^2}{2}$

    (B) $\dfrac{1 + e^2}{4}$

    (C) $\dfrac{2 + e^2}{2}$

    (D) $\dfrac{4 + e^2}{4}$

68. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1 + x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \dfrac{1}{\sqrt{2}} \right)^{200}$. If $\dfrac{^{200}C_{99}K}{a} = \dfrac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to

    (A) (50, 51) (B) (50, 101)
    (C) (51, 99) (D) (51, 101)

69. The area of the region $A = \left\{ (x, y) : |\cos x - \sin x| \le y \le \sin x, 0 \le x \le \dfrac{\pi}{2} \right\}$ is

    (A) $\sqrt{5} + 2\sqrt{2} - 4.5$

    (B) $\dfrac{3}{\sqrt{5}} - \dfrac{3}{\sqrt{2}} + 1$

    (C) $\sqrt{5} - 2\sqrt{2} + 1$

    (D) $1 - \dfrac{3}{\sqrt{2}} + \dfrac{4}{\sqrt{5}}$

70. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

    (A) 79 (B) 89
    (C) 84 (D) 86

71. If the lines $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z+3}{1}$ and $\dfrac{x-a}{2} = \dfrac{y+2}{3} = \dfrac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane z = a is:

    (A) 16 (B) 10
    (C) 22 (D) 28

72. The value of the integral $\displaystyle\int_{\frac{1}{2}}^{2} \dfrac{\tan^{-1} x}{x} \, dx$ is equal to

    (A) $\dfrac{\pi}{4} \log_e 2$

    (B) $\dfrac{1}{2} \log_e 2$

    (C) $\pi \log_e 2$

    (D) $\dfrac{\pi}{2} \log_e 2$

73. Let S = $\{w_1, w_2, \ldots\}$ be the sample space associated to a random experiment. Let $P(w_n) = \dfrac{P(w_{n-1})}{2}$, $n \ge 2$. Let A = $(2k + 3l : k, l \in N)$ and B = $\{W_n : n \in A\}$. Then P(B) is equal to

    (A) $\dfrac{1}{32}$

    (B) $\dfrac{3}{64}$

    (C) $\dfrac{3}{32}$

    (D) $\dfrac{1}{16}$

74. The plane $2x - y + z = 4$ intersects the line segment joining the points A(a, –2, 4) and B(2, b, –3) at the point C in the ratio 2:1 and the distance of the point C from the origin is $\sqrt{5}$. If ab < 0 and P is the point (a – b, b, 2b – a) then CP^2 is equal to

    (A) $\dfrac{16}{3}$

    (B) $\dfrac{17}{3}$

    (C) $\dfrac{97}{3}$

    (D) $\dfrac{73}{3}$

75. Consider a function f: $\mathbb{N} \to \mathbb{R}$, satisfying f(1) + 2f(2) + 3f(3) +…+ xf(x) = x(x + 1) f(x); $x \ge 2$ with f(1) = 1. Then $\dfrac{1}{f(2022)} + \dfrac{1}{f(2028)}$ is equal to

    (A) 8400 (B) 8000
    (C) 8100 (D) 8200

76. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is

    (A) $\left[-1, -\dfrac{1}{2} \right]$

    (B) $[-2, -1]$

    (C) $\left[-\dfrac{3}{2}, -1 \right]$

    (D) $\left[-2, -\dfrac{3}{2} \right]$

77. Let f and g be twice differentiable functions on R such that
$f''(x) = g''(x) + 6x$
$f'(1) = 4g'(1) - 3 = 9$
$f(2) = 3g(2) = 12.$
Then which of the following is **NOT** true?

(A) There exists $x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = g(x_0)$

(B) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(C) $g(-2) - f(-2) = 20$

(D) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

78. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{5}{\sqrt{2}}$ (C) $\frac{1}{5}$ (D) $\frac{3}{\sqrt{2}}$

79. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to:
(A) $B \Rightarrow (A \Rightarrow B)$
(B) $A \Rightarrow ((\sim A) \Rightarrow B)$
(C) $A \Rightarrow (A \Leftrightarrow B)$
(D) $B \Rightarrow ((\sim A) \Rightarrow B)$

80. The shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$ is
(A) $5\sqrt{3}$ (B) $2\sqrt{3}$
(C) $3\sqrt{3}$ (D) $4\sqrt{3}$

Section – B

Numerical Value Type Questions

81. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$ and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _____.

82. A circle with centre (2, 3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to _____.

83. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is _____.

84. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A - \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____.

85. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point (1, –3) is $x - 4y = 13$, then the value of a + b is equal to _____

86. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall \ n \geq 2$. If $S = \sum_{n-1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to _____.

87. Let $\alpha_1, \alpha_2, ..., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq ... \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to _____

88. Let $X = \{11, 12, 13, ..., 40, 41\}$ and $Y = \{61, 62, 63, ..., 90, 91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $|\bar{x} + \bar{y} - \sigma^2|$ is equal to _____.

89. Let $\alpha = 8 - 14i$, $A = \left\{ z \in C : \frac{\alpha z - \bar{\alpha} \bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1 \right\}$ and $B = \{z \in C : |z + 3i| = 4\}$. Then $\sum_{z \in A \cap B} (\text{Re} z - \text{Im} z)$ is equal to _____.

90. Let $\{a_k\}$ and $\{b_k\}$, $k \in N$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in N$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to _____.